2

Theory of Simplified Sewerage

This Section presents the theory of simplified sewer design. Firstly, in Section 2.1, the peak daily wastewater flow in the length of sewer being designed is described. Section 2.2 presents the trigonometric properties of a circular section, as the sewers used in simplified sewerage are of circular cross-section. The Gauckler-Manning equation for the velocity of flow in a sewer and the corresponding flow equation are given in Section 2.3. Tractive tension is described in Section 2.4, and the minimum sewer gradient based on the design minimum tractive tension is derived in Section 2.5. The procedure for calculating the sewer diameter is given in Section 2.6, and that for determining the maximum number of houses served by a sewer of given diameter in Section 2.7. Finally, the results of a simplified sewer design trial are presented in Section 2.8; this was a comparison of designs based on the Gauckler-Manning, Colebrook-White and Escritt equations (details are given in Appendices 1 and 2). The overall design procedure follows that given in Mara (1996) (see also, Yao, 1974; Machado Neto and Tsutiya, 1985; de Melo, 1985 and 1994; Bakalian *et al.*, 1994).

2.1 WASTEWATER FLOW

The value of the wastewater flow used for sewer design is the daily peak flow. This can be estimated as follows:

$$q = k_1 \ k_2 \ Pw \ / \ 86 \ 400 \qquad\qquad (2.1)$$

where q = daily peak flow, l/s
k_1 = peak factor (= daily peak flow divided by average daily flow)
k_2 = return factor (= wastewater flow divided by water consumption)
P = population served by length of sewer under consideration
w = average water consumption, litres per person per day

and 86 400 is the number of seconds in a day.

A suitable design value for k_1 for simplified sewerage is 1.8 and k_2 may be taken as 0.85. Thus equation 2.1 becomes:

$$q = 1.8 \times 10^{-5} \ Pw \qquad\qquad (2.2)$$

The design values given above for the peak flow factor, k_1 and the return factor, k_2 (1.8 and 0.85 respectively) have been found to be suitable in Brazil, but they may

need changing to suit conditions elsewhere – especially if stormwater (for example, roof drainage water) is discharged into the simplified sewer. However, this should **not** be permitted to occur as the resulting design for what is in practice partially combined sewerage system would be based on a much higher value for k_1 (perhaps as high as 3 or 4), but see Section 2.1.1.

Variations in the value of k_2 have a much lower impact on design, except in middle- and high-income areas where a large proportion of water consumption is used for lawn-watering and car-washing. In periurban areas in Brazil a k_2 value of 0.85 has been used successfully, although CAESB now uses a value of 0.65, even in low-income areas and without any reported operational problems (Luduvice, 2000). However higher values may be more appropriate elsewhere – for example, in areas where the water supply is based on a system of public standpipes, values up to 0.95 may be used.

2.1.1 Minimum daily peak flow

In simplified sewer design equation 2.1 or 2.2 is used to calculate the daily peak flow in the length of sewer under consideration, but subject to a minimum value of 1.5 l/s (see Section 2.6). This minimum flow is not justifiable in theory but, as it is approximately equal to the peak flow resulting from flushing a WC, it gives sensible results in practice, and it is the value recommended in the current Brazilian sewer design code (see ABNT, 1986; also Sinnatamby, 1986, although he used a minimum flow of 2.2 l/s).

With the use of this minimum value for the peak daily flow, the values used for k_1 and k_2 in equation 2.1 become less important, especially for short lengths of sewer. For example, for a length of sewer serving 500 people with a water consumption of 80 litres per person per day and using a return factor of 0.85, the average daily wastewater flow is given by equation 2.3 as:

$$q = k_2 \, Pw / 86\,400 \tag{2.3}$$

$$= 0.85 \times 500 \times 80 / 86\,400$$

$$= 0.4 \text{ l/s}$$

For the minimum peak daily flow of 1.5 l/s, this is equivalent to a k_1 value of (1.5/0.4) = 3.75. Thus for condominial sewers serving even quite a large number of people, there is an inherent allowance for at least some stormwater (see Section 3.3.3).

2.2 PROPERTIES OF A CIRCULAR SECTION

The flow in simplified sewers is always open channel flow – that is to say, there is always some free space above the flow of wastewater in the sewer. The hydraulic design of simplified sewers requires knowledge of the area of flow and the hydraulic radius. Both these parameters vary with the depth of flow, as shown in Figure 2.1. From this figure, trigonometric relationships can be derived for the following parameters:

(1)   the **area of flow** (a), expressed in m^2;

(2)   the **wetted perimeter** (p), m;

(3)   the **hydraulic radius** (r), m; and

(4)   the **breadth of flow** (b), m.

The hydraulic radius (sometimes called the hydraulic mean depth) is the area of flow divided by the wetted perimeter.

The breadth of flow is used for the calculation of the risk of hydrogen sulphide generation (see Appendix 3), and also in Escritt's (1984) definition of hydraulic radius (see Section A1.4 in Appendix 1).

Parameters 1 – 4 above depend on the following three parameters:

(5)   the **angle of flow** (θ), expressed in *radians*;

(6)   the **depth of flow** (d), m; and

(7)   the **sewer diameter** (D), m.

If the angle of flow is measured in degrees, then it must be converted to radians by multiplying by ($2\pi/360$), since 360° equals 2π radians.
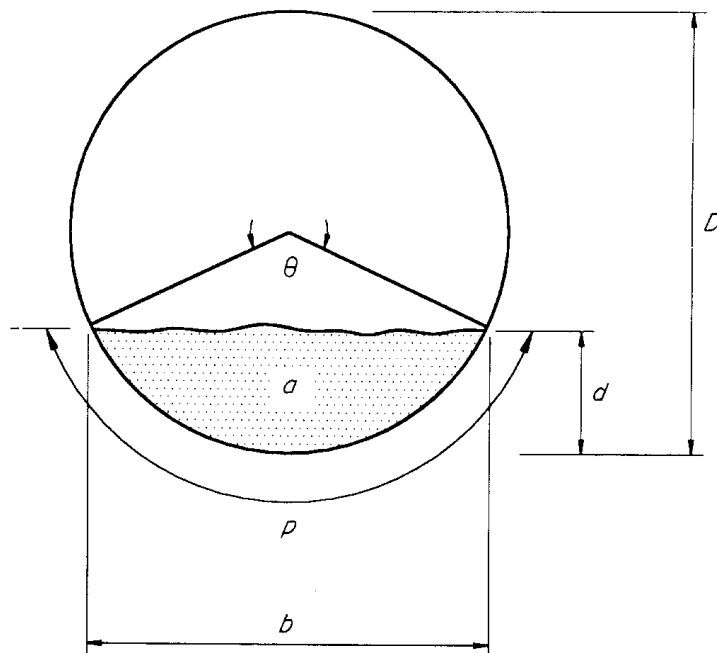

Figure 2.1 Definition of parameters for open channel flow in a circular sewer. *Source*: Mara (1996).

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The ratio d/D is termed the **proportional depth of flow** (which is dimensionless). In simplified sewerage the usual limits for d/D are as follows:

$$0.2 < d/D < 0.8$$

The lower limit ensures that there is sufficient velocity of flow to prevent solids deposition in the initial part of the design period, and the upper limit provides for sufficient ventilation at the end of the design period.

The equations are as follows:

(a) **Angle of flow:**

$$\theta = 2 \cos^{-1} [1 - 2\,(d/D)] \qquad\qquad\qquad (2.4)$$

(b) **Area of flow:**

$$a = D^2\, [(\theta - \sin \theta)\, /\, 8] \qquad\qquad\qquad (2.5)$$

(c) **Wetted perimeter:**

$$p = \theta\, D/2 \qquad\qquad\qquad (2.6)$$

(d) **Hydraulic radius** (= a/p):

$$r = (D/4)\, [1 - ((\sin \theta)\, /\theta)] \qquad\qquad\qquad (2.7)$$

(e) **Breadth of flow**:

$$b = D \sin (\theta/2) \qquad\qquad\qquad (2.8)$$

When $d = D$ (that is, when the sewer is flowing just flow), then $a = A = \pi\, D^2/4$; $p = P = \pi D$ and $r = R = D/4$.

The following equations for a and r are used in designing simplified sewers:

$$a = k_a D^2 \qquad\qquad\qquad (2.9)$$

$$r = k_r D \qquad\qquad\qquad (2.10)$$

The coefficients k_a and k_r are given from equations 2.5 and 2.6 as:

$$k_a = \frac{1}{8}(\theta - \sin\theta) \qquad\qquad\qquad (2.11)$$

$$k_r = \frac{1}{4}[1 - ((\sin\theta\,/\,\theta)] \qquad\qquad\qquad (2.12)$$

When $a = A$ and $r = R$, then $k_a = \pi/4$ and $k_r = 0.25$.

2.3 GAUCKLER-MANNING EQUATION

In 1889 Robert Manning (an Irish civil engineer, 1816-1897) presented his formula relating the velocity of flow in a sewer to the sewer gradient and the hydraulic radius (Manning, 1890). The formula is commonly, but improperly, known as the Manning equation; as pointed out by Williams (1970) and Chanson (1999), it should be known as the Gauckler-Manning equation since Philippe Gauckler (a French civil engineer, 1826-1905) published the same equation 22 years earlier (Gauckler, 1867 and 1868). The Gauckler-Manning equation (see Appendix 1) is:

$$v = (1/n)\ r^{2/3}\ i^{1/2} \tag{2.13}$$

where v = velocity of flow at d/D, m/s
   n = Ganguillet-Kutter roughness coefficient, dimensionless (but see Appendix 1)
   r = hydraulic radius at d/D, m
   i = sewer gradient, m/m (i.e. dimensionless)

Since flow = area $\times$ velocity,

$$q = (1/n)\ a\ r^{2/3}\ i^{1/2} \tag{2.14}$$

where q = flow in sewer at d/D, m^3/s

Using equations 2.9 and 2.10, equation 2.14 becomes:

$$q = (1/n)\ k_a\ D^2\ (k_r\ D)^{2/3}\ i^{1/2} \tag{2.15}$$

The usual design value of the Ganguillet-Kutter roughness coefficient, n is 0.013. This value is used for any relatively smooth sewer pipe material (concrete, PVC or vitrified clay – but see Appendix 3) as it depends not so much on the roughness of the material itself, but on the roughness of the bacterial slime layer which grows on the sewer wall.

2.4 TRACTIVE TENSION

Tractive tension (or boundary shear stress) is the tangential force exerted by the flow of wastewater per unit wetted boundary area. It is denoted by the symbol τ (the Greek letter *tau*) and has units of N/m^2 (i.e. Pascals, Pa). As shown in Figure 2.2, and considering a mass of wastewater of length l m and cross-sectional area a m^2, which has a wetted perimeter of p m, the tractive tension is given by the component of the weight (W, Newtons) of this mass of wastewater in the direction of flow divided by its corresponding wetted boundary area (i.e. the area in which it is in contact with the sewer = pl):

$$\tau = W \sin \phi\ /\ pl \tag{2.16}$$

The weight W is given by:

$$W = \rho g a l \qquad (2.17)$$

where ρ = density of wastewater, kg/m^3
$\qquad$ g = acceleration due to gravity, m/s^2

So that, since a/p is the hydraulic radius, r:

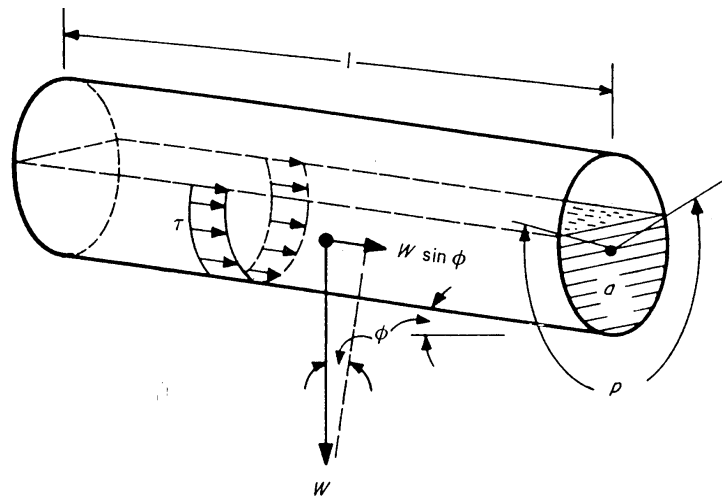$$\tau = \rho g r \sin \phi \qquad (2.18)$$

Figure 2.2 Definition of parameters for tractive tension in a circular sewer. *Source*: Barnes *et al.* (1981).

When ϕ is small, $\sin \phi = \tan \phi$, and $\tan \phi$ is the sewer gradient, i (m/m). Thus, equation 2.18 can be rewritten as:

$$\tau = \rho g r i \qquad (2.19)$$

Using equation 2.10 and rearranging:

$$D = (\tau/\rho g) / k_r i \qquad (2.20)$$

Substituting this expression for D in equation 2.15 and simplifying:

$$q = (1/n) \, k_a \, k_r^{-2} \, (\tau / \rho g)^{8/3} \, i^{-13/6} \qquad (2.21)$$

2.5 MINIMUM SEWER GRADIENT

The minimum sewer gradient, I_{min} is given by rearranging equation 2.21 and substituting I_{min} for i and τ_{min} for τ, as follows:

$$I_{min} = [(1/n) \, k_a \, k_r^{-2}]^{6/13} \, [\tau_{min} / \rho g]^{16/13} \, q^{-6/13} \qquad (2.22)$$

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For $d/D = 0.2$, the minimum value used in simplified sewerage – that is, from equations 2.4, 2.11 and 2.12, for $k_a = 0.1118$ and $k_r = 0.1206$; and with $n = 0.013$, $\rho = 1000$ kg/m^3 and $g = 9.81$ m/s^2, equation 2.22 becomes:

$$I_{min} = 2.33 \times 10^{-4} \, (\tau_{min})^{16/13} \, q^{-6/13} \qquad (2.23)$$

A good design value for τ_{min} in simplified sewerage is 1 Pa; thus:

$$I_{min} = 2.33 \times 10^{-4} \, q^{-6/13} \qquad (2.24)$$

In this equation the units of q are m^3/s. Changing them to litres/second gives:

$$I_{min} = 5.64 \times 10^{-3} \, q^{-6/13} \qquad (2.25)$$

Equations 2.24 and 2.25 are for a value of τ_{min} of 1 Pa. This value has been successfully used in simplified sewerage systems in southern Brazil where the systems are wholly separate – PVC pipes are used and junction boxes (see Section 5.1.5) are either plastic or, if in brick, have their coverslab well mortared on; thus the ingress of stormwater, soil, grit etc. into the sewer is minimal; moreover used toilet paper is commonly not disposed of in the toilet bowl, but into an adjacent bucket for disposal with household garbage. Yao (1974) recommends values of τ_{min} for sanitary sewers of 1-2 Pa, and 3-4 Pa for stormwater or combined sewers. Designers must make an appropriate choice for τ_{min}, and use equation 2.23 for values > 1 Pa. Values of τ_{min} > 1 Pa have a large influence on the value of I_{min}. For example, for a flow of 1.5 l/s, equation 2.23 gives:

τ_{min} (Pa)	I_{min}
1	1 in 213
1.5	1 in 130
2	1 in 91

CAESB, the water and sewerage company of Brasília and the Federal District, uses a τ_{min} of 1 Pa and a minimum value of I_{min} of 0.5% (1 in 200). In low-income areas this has not resulted in any significant operational problems (Luduvice, 2000).

2.6 SEWER DIAMETER

Equation 2.15 can be rearranged, as follows, writing $i = I_{min}$:

$$D = n^{3/8} \, k_a^{-3/8} \, k_r^{-1/4} \, (q/I_{min}^{1/2})^{3/8} \qquad (2.26)$$

In this equation the units of D are m, and the units of q are m^3/s.

The sewer diameter is determined by the following sequence of calculations:

(1)   Calculate using equation 2.2, the initial and final wastewater flows (q_i and q_f, respectively, in l/s), which are the flows occurring at the start and end of the

design period. (The increase in flow is due either to an increase in population or an increase in water consumption, or both.)

If the flow so calculated is less than the minimum peak daily flow of 1.5 l/s (see Section 2.1.1), then use in (2) below a value of 1.5 l/s for q_i.

(2)   Calculate I_{min} from equation 2.25 with $q = q_i$.

(3)   Calculate D from equation 2.26 using $q = q_f$ (in m³/s), again subject to a minimum value of 0.0015 m³/s, for $d/D = 0.8$ (i.e. for $k_a = 0.6736$ and $k_r = 0.3042$ from equations 2.4, 2.11 and 2.12).

In this design procedure, the value of q_i is used to determine I_{min} and the value of q_f is used to determine D (Box 1.1).

The diameter so calculated is unlikely to be a commercially available size, and therefore the next larger diameter that is available is chosen (i.e. if $D = 86$ mm, say, then choose 100 mm).

The **minimum diameter** used in simplified sewerage is 100 mm (but see Section 3.3.4).

2.7   NUMBER OF HOUSES SERVED

In the detailed design of condominial sewers (Section 3.2.6) it is useful to know the maximum number of houses that can be served by a sewer of given diameter. The procedure for calculating this is shown here – *as an example only* – for a household size of 5, a per caput water consumption of 100 l/d, a peak factor of 1.8 and a return factor of 0.85. The peak flow per household (q_h, l/s) is given by equation 2.2 as:

$$q_h = 1.8 \times 10^{-5} \, P \, w$$

$$= 1.8 \times 10^{-5} \times 5 \times 100$$

$$= 0.009 \text{ l/s per household.}$$

If it is assumed that the housing area is fully developed (i.e. that there is no space for further houses), then any increase in wastewater flow will be due to an increase in water consumption.

Designing the sewer for an initial d/D of 0.6, allows for an increase in water consumption to just under 150 litres per caput per day when d/D will be the maximum value of 0.8 (see Mara, 1996) – such an increase is more than adquate.

Equations 2.15 (with $i = I_{min}$) and 2.22 are now solved for $d/D = 0.6$ (i.e. for $k_a = 0.4920$ and $k_r = 0.2776$), with $\tau_{min} = 1$ Pa and with q in l/s, as follows:

$$I_{min} = 0.00518 \, q^{-6/13} \tag{2.27}$$

$$D = 0.0264 \, (q/I_{min}^{1/2})^{3/8} \tag{2.28}$$

Thus, with D in mm:

$$q = 9.8 \times 10^{-5} \, D^{13/6} \tag{2.29}$$

The peak flow per household is 0.009 l/s, so that q is given by:

$$q = 0.009 \, N \tag{2.30}$$

where N = number of houses served. Thus:

$$N = 10.89 \times 10^{-3} \, D^{13/6} \tag{2.31}$$

Equation 2.31 shows that, *for the design values assumed*, a 100 mm diameter sewer can serve up to 234 houses. For any other set of design parameters (including the initial value of d/D) an equation corresponding to equation 2.31 has to be derived in the manner shown above.

2.8 DESIGN COMPARISONS

In Sections 2.3 – 2.6 the **Gauckler-Manning** equation was used to exemplify the basis of the hydraulic design of simplified sewers. Although it is the only equation to have been used to date for simplified sewer design in practice, there are two other principal equations which are currently used for the hydraulic design of conventional sewers, and which could in principle therefore be used for simplified sewer design. They are:

(1) the **Colebrook-White** equation (Colebrook, 1938; see also Butler and Pinkerton, 1987 and HR Wallingford and Barr, 1994), and

(2) the **Escritt** equation (Escritt, 1984).

Appendix 1 presents an overview of the development of the Gauckler-Manning, Colebrook-White and Escritt equations, and Appendix 2 contains the results of comparative trials using these three equations to identify which one is the most suitable overall. These trials comprised comparisons based on the simplified sewer design examples given in Sinnatamby (1986) and Bakalian *et al*. (1994). The results of these trials, show that there is no advantage in using either the Colebrook-White equation or the Escritt equation over the Gauckler-Manning equation. The latter is therefore preferred for use in the PC-based design of simplified sewers detailed in Section 4, although the program allows any of the three to used.