

# Formulation of biomass production model using equations of primary growth factors and environmental database for selected groups of aquatic macrophytes

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**Abstract** A simple mathematical model for biomass production was formulated by incorporating the equations of primary growth factors into the logistic growth model. The model simulated biomass of aquatic macrophytes *viz.* floating macrophytes Water hyacinth (*Eichhornia crassipes* (Mart.) Solms) and Giant duckweed (*Spirodela polyrhiza*) and; emergent macrophytes Purple loosestrife (*Lythrum salicaria*) and Common reed (*Phragmites australis*) was within  $\pm 12\%$  discrepancy. The structure of the model is simple, parameters are easy to measure and input data can be obtained from public domains. Therefore, the model structure can be applied to other groups of macrophytes.

**Keywords** Common reed; growth factors; limiting function; logistic growth; Yamanashi

## INTRODUCTION

Physical environmental factors drive growth of autotroph like plants. Many growth and biomass production models (Mahujchariyawong and Ikeda, 2001; Herb and Stefan, 2003) reported are based on physical factors and physiological processes or only either of them and are mostly complex in structure and data intensive. Some single physical factor like temperature dependent growth equations (Yan and Hunt, 1999; van der Heide *et al.*, 2006) or more physical factors like light and temperature (Gopal, 1987) and light and nutrient (Lorenti *et al.*, 1995) dependent growth models have also been reported. However, growth and biomass production model using all major physical environmental factors have received less attention. Therefore, the present study aims to formulate a simple mathematical biomass production model based on major environmental physical factors and; apply the model in selected groups of aquatic macrophytes. The model of the present study has scope for environmental engineers to advance research into environmental factors that regulate growth and predict growth and biomass changes over time under variable environmental scenarios. For lake and pond management, the model is simple especially to prepare biomass harvesting schedules for the maintenance of biomass at optimum levels.

## METHODS

### Model formulation

The model in this study was based on conventional model with simple logistic growth function (Equation 1). In equation 1,  $dB/dt$  is the growth rate of plants dependent on their specific growth rate,  $G$ , maximum biomass carrying capacity,  $B_{\max}$ , and available biomass,  $B$ , ( $0 \leq B \leq B_{\max}$ ). In addition,  $G$  is the product of maximum specific growth rate ( $G_{\max}$ ) and limiting functions of primary growth factors like temperature, light, nutrient and mortality, a negative growth factor ( $0 \leq$  growth factors  $1 \leq$ ). The solution of equation 1 leads to equation 2.

$$dB / dt = G \times B(1 - B / B_{\max}) \quad (1)$$

$$B = B_{\max} / (1 + C \times \exp(-G \times t)) \text{ where } C = (B_{\max} - B) / B \quad (2)$$

Growth of macrophytes observed as a function of primary growth factors were compared with some equation types of growth factors (Figure 1). The equation types that closely reproduced the observed growth of macrophytes were incorporated into the biomass production model to be formulated as equations of growth factors. The equations were solved as growth limiting functions ( $0 \leq \text{growth factors} \leq 1$ ) to evaluate optimal environmental conditions prevailed during the growth of test plants *viz.* floating macrophytes Water hyacinth (*Eichhornia crassipes* (Mart.) Solms) and Giant duckweed (*Spirodela polyrhiza*) and; emergent macrophytes Purple loosestrife (*Lythrum salicaria*) and Common reed (*Phragmites australis*).

### Experimental set up

Laboratory experiment was set up to measure growth parameters of test plants under hydroponic system using 1/10 Hutner (Hutner, 1953) solution. Lineweaver-Burk plot was used for determining semi-saturation constant of total nitrogen (T-N) and total phosphorus (T-P) and mortality constants were estimated by least square method. Standard Methods of Analysis (APHA 1975) was followed for analysis of T-N and T-P of water. Temperature and light intensity parameters were measured for test plants, while input data was obtained from Japan Meteorological Agency. Field experiment was carried out in the pond located in Yamanashi Prefectural Wood Park at Kanegawa. The biomass and water nutrient data for the test plants and water bodies were collected from the same test pond. The floating wooden frame (1x1m<sup>2</sup>) was used as an experimental base filled with coir to thickness of 15cm and the biomass density of test plants was maintained as necessary.

## RESULTS AND DISCUSSION

### Selection of equations of primary growth factors

Arrhenius type (Herb, 2003; Soetaert, 2004) and  $\beta$ -distribution type (Collins, 1989; Yan and Hunt, 1999; Buonomo *et al.*, 2005) models are widely used to model the relationship between growth and temperature. In both type of formulation (i.e.  $\beta$ -distribution and Arrhenius type) the growth rate exponentially increases with increase in temperature until it reaches the optimal temperature. However,  $\beta$ -distribution type can model the decrease in growth rate after temperature increases further from the optimum temperature (Figure 1a). The  $\beta$ -distribution type of Yan and Hunt (1999) was selected as it performed better in reproducing growth and temperature relationship as compared to other distribution type. Michaelis-Menten equation type (Asaeda, 2000; Herb, 2003; Soetaert, 2004) and Steele (1962) type (Collins, 1989; Buonomo *et al.*, 2005) models are widely used to model the relationship between growth and light intensity. In both models, the growth rate increases rectilinearly with light intensity and it becomes slower as light intensity reaches near saturation. In addition to this Steele type formulation also reproduced decreasing growth rate trend of macrophytes after their saturation growth and so it was selected (Figure 1b). Michaelis-Menten equation types is a widely used nutrition salt density model (Mahujchariyawong and Ikeda, 2001; Buonomo *et al.*, 2005). This model shows rectilinear increase in growth rate with increase in nutrition salt density (Figure 1c). Along with this model, simultaneous use of concepts of Liebig's Law of Minimum is also relevant as both N and P present in water bodies influence plant growth (when  $N/K_N < P/K_P$  or when  $P/K_P < N/K_N$ ). Mortality has negative influence on growth. The

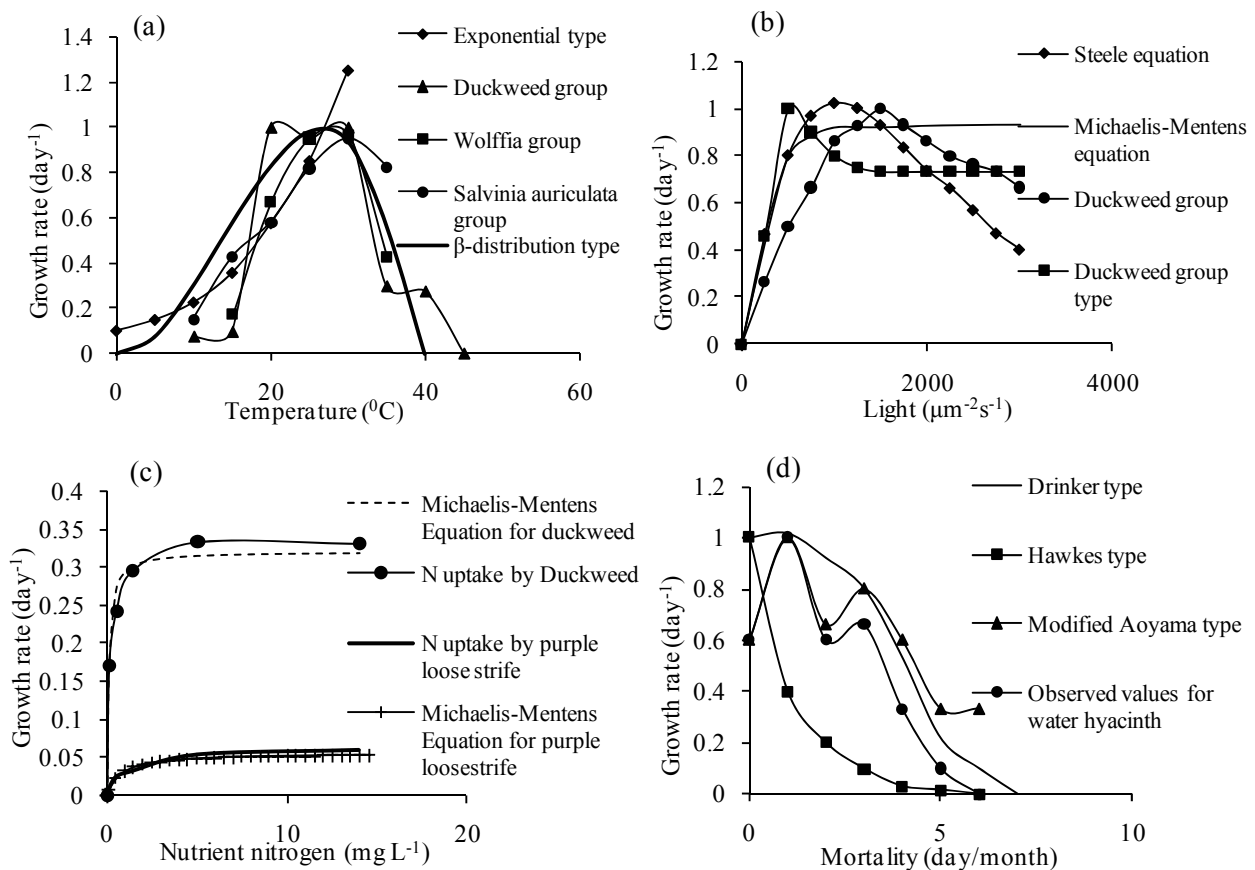
exponential decrease of growth rate was in about 2 months in actual observed data of Water hyacinth by Aoyama (1986) and observed data of Water hyacinth of this study, while it was in the early days of the first month in Hawkes type (2000) (Figure 1d). The Drinker type nullified the influence of mortality of the first month plant growth which was not in case of Hawkes type and model results of Hawkes type varied largely from the actual observed data. To sum up, the equation types selected are given as a group of equation 3 which were then incorporated in equation 2 as G.

$$T_1 = (T_{max} - T / T_{max} - T_{opt}) (T / T_{opt})^{T_{opt} / (T_{max} - T_{opt})}; L = I / I_0 \times \exp[1 - (I / I_0)]; \quad (3)$$

$$N = C_n / (K_n + C_n); P = C_p / (K_p + C_p) \text{ and}$$

$$M = 1 - 1 / (1 + b \times \exp(-c \times t))$$

In equation 3,  $L$  is light intensity ( $\text{Wm}^{-2}$ ),  $I$  is observed light intensity ( $\text{mJm}^{-2}$ ),  $I_0$  is optimum light intensity ( $\text{mJm}^{-2}$ ),  $N$  and  $P$  are concentration of N and P ( $\text{mgL}^{-1}$ ),  $C_n$  and  $C_p$  are underwater concentration of T-N and T-P respectively ( $\text{mgL}^{-1}$ ),  $K_n$  and  $K_p$  are semi-saturation constant for T-N and T-P respectively ( $\text{mgL}^{-1}$ ),  $T$  is ambient temperature ( $^{\circ}\text{C}$ ),  $T_{max}$  is maximum temperature ( $^{\circ}\text{C}$ ) and  $T_{opt}$  is optimum temperature ( $^{\circ}\text{C}$ ),  $M$  is mortality,  $b$  and  $c$  are constant and  $t$  is time (days).



**Figure 1** Comparison of growth of macrophytes and primary growth factor equations (a) temperature (b) light (c) nutrient uptake and (d) mortality

### Growth parameters

Most of the parameters of the model were estimated through measurement instead of model calibration (Table 1). The maximum biomass of Water hyacinth was about 2.5, 6 and 24 times more

than Purple loosestrife, Common reed and Giant duckweed respectively. Likewise, the growth rate of Giant duckweed was three times of Water hyacinth and Common reed and two times of Purple loosestrife. High biomass and growth rates of macrophytes are valuable for wastewater purification. Maximum and optimum temperatures and light intensities values suggest that the macrophytes grow well in warm environment. Semi-saturation constant for T-N of Purple loosestrife was more than eight times than Water hyacinth, Giant duckweed and Common reed respectively; while T-P was more than six times than Water hyacinth and Giant duckweed and three times than Common reed suggesting their high nutrient assimilating ability. Measuring optimal light intensity for plants equivalent to natural conditions was not reasonable in the greenhouse chamber of laboratory (maximum  $10.6 \text{ Wm}^{-2}$ ). For this reason, light intensity of Kofu, Yamanashi ( $411.0 \text{ Wm}^{-2}$ ) when maximum growth of Water hyacinth was observed in the test pond was considered its optimal light intensity as well as for other emergent macrophytes under study. Furthermore, modeling results revealed that the modeling results are relatively insensitive to optimum light condition for all macrophytes specified above. However, this value of optimum light intensity parameter needs to be estimated through model calibration or through measurement if any area has light intensity of wide ranges. Laboratory greenhouse chamber measured optimum light intensity of Giant duckweed was  $10.2 \text{ Wm}^{-2}$  when its maximum growth rate was  $0.32 \text{ day}^{-1}$ . Interestingly, Vermaat and Hanif, 1998 too reported similar value of growth rate for Giant duckweed grown in natural environment where light intensities were high. Moreover, modeling results for both Giant duckweeds grown in natural water bodies and inside laboratory greenhouse chamber were virtually indistinguishable thereby suggesting the ability of Giant duckweed to grow in wide range of light intensities.

**Table 1** Parameters of the growth model for some groups of macrophytes

Parameter	Unit	Water hyacinth	Giant Duckweed	Purple loosestrife	Common reed
Maximum biomass ( $B_{\max}$ )	$\text{Kg-FWm}^{-2}$	70.0 <sup>a</sup>	2.96*	29.16*	12.2*
Maximum specific growth rate ( $G_{\max}$ )	$\text{day}^{-1}$	0.12 <sup>b,c</sup>	0.32*	0.21*	0.07*
Optimum temperature ( $T_0$ )	$^{\circ}\text{C}$	29.0 <sup>c, d</sup>	27.0 <sup>c, d</sup>	27.5*	25.0*
Maximum temperature ( $T_{\max}$ )	$^{\circ}\text{C}$	40.0 <sup>c, d</sup>	40.0 <sup>c, d</sup>	40.0*	40.0*
Optimum amount of light ( $L_0$ )	$\text{Wm}^{-2}$	411.0*	10.22*	411.0*	411.0*
Semi saturation constant for N ( $K_N$ )	$\text{mg T-NL}^{-1}$	0.06 <sup>a, c</sup>	0.089*	0.752*	0.082*
Semi saturation constant for P ( $K_P$ )	$\text{mg T-PL}^{-1}$	0.009 <sup>a, c</sup>	0.008*	0.056*	0.02*
Mortality constant (a)	-	4.6 <sup>e</sup>	14.5 <sup>e</sup>	4.0 <sup>e</sup>	135.0 <sup>e</sup>
Mortality constant (b)	-	1.20 <sup>e</sup>	1.23 <sup>e</sup>	0.01 <sup>e</sup>	1.23 <sup>e</sup>

\* Measured; <sup>a</sup> Wilson *et al.* 2005; <sup>b</sup>Lissner *et al.* 1999; <sup>c</sup>Lorber *et al.* 1984; <sup>d</sup>Song *et al.* 2006; <sup>e</sup>estimated

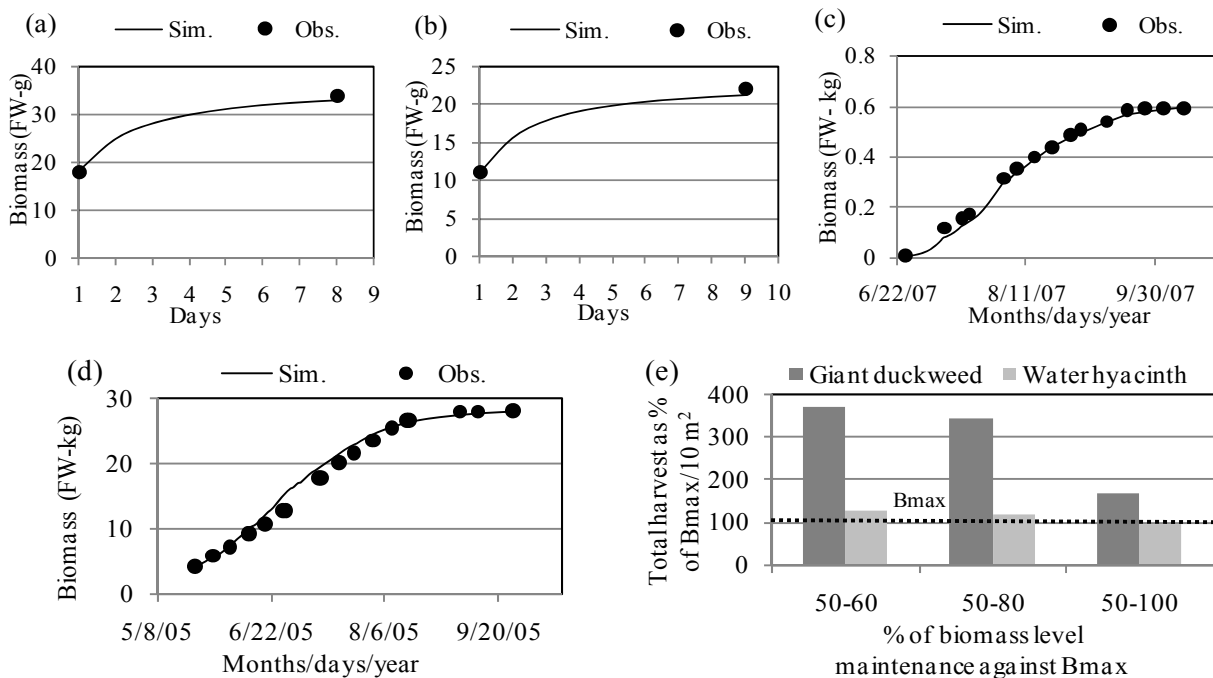
### Model simulated biomass production of macrophytes

Average monthly environmental conditions initially at slow, rapid and steady phases of growth of macrophytes, limiting function number of growth factors and model simulated cumulative biomass and growth trends during the growth of macrophytes under study are presented in this section.

#### *Biomass production by floating macrophytes*

Giant duckweed was cultivated in laboratory under short time cultivation tests for twice each of about one week (Figure 2a and 2b). In both the tests, model simulations were close (4% discrepancies) to the observed biomass productions. The cultivation condition in laboratory was almost constant with temperature of  $25^{\circ}\text{C}$ , light intensity of  $10.2 \text{ Wm}^{-2}$  and minimum nutrient T-P (minimum nutrient according to Liebig's Law of Minimum) ranging from 0.54 to  $1.62 \text{ mg T-PL}^{-1}$ . However, growth limiting function number for temperature and nutrient were 0.99 and for light

intensity it was 0.90. Likewise, environmental conditions prevailed for Giant duckweed under long term cultivation test of 3.5 months (Figure 2c) was 24°C in May-June, 27.7°C in August and 24.7°C in September (optimal 27°C); light intensity was 333.3 Wm<sup>-2</sup> except it was 416.6 Wm<sup>-2</sup> in August (optimal 10.2 Wm<sup>-2</sup> for laboratory condition) and; T-P varied between 0.094 to 0.12 mg T-PL<sup>-1</sup> (semi-saturation constant 0.008 mg T-PL<sup>-1</sup>). Respective limiting function number for temperature, light and nutrient were 0.94, 1.0 and 0.78. The model simulated the observed biomass (0.59 kg) with no apparent discrepancy and matching the growth trend. Environmental conditions during long term cultivation (4 months) of Water hyacinth (Figure 2d) was on an average 21, 27 and 24°C in May, August and September respectively (optimal 29°C); light intensity was 455.5, 408.3 and 291.6 Wm<sup>-2</sup> (optimal 411.0 Wm<sup>-2</sup>) and; T-P in the pond was 0.05 mg T-PL<sup>-1</sup>(semi-saturation constant 0.009 mg T-PL<sup>-1</sup>) in the same period. Limiting function number for temperature, light and nutrient were 0.90, 0.92 and 0.85 respectively. The model simulated the observed biomass and the growth trend with no apparent discrepancy.

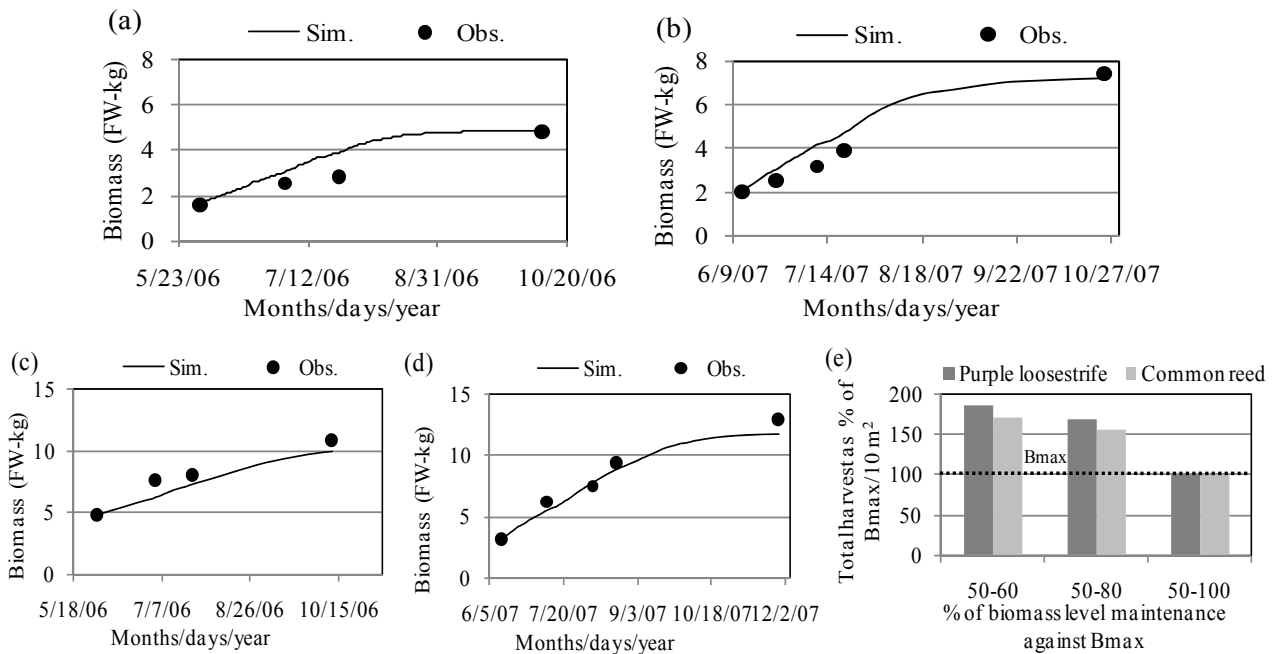


**Figure 2** Comparison between simulated and observed biomass (a) and (b) Giant duckweed (short term cultivation test), (c) Giant duckweed (long term cultivation test), (d) Water hyacinth (long term cultivation test) and; (e) an illustration of total biomass harvest against different level of biomass maintenance ( $B_{max}$  is considered as maximum biomass/m<sup>2</sup> produced by the macrophytes under natural condition)

### *Biomass production by emergent macrophytes*

Long term cultivation test (4-5 months) for Purple loosestrife and Common reed was conducted in 2006 and 2007. The temperature for Purple loosestrife in 2006 was 22, 27 and 18°C (optimal 27.5°C); light intensity was 361.1, 444.4 and 305.5 Wm<sup>-2</sup> (optimal 411.0 Wm<sup>-2</sup>) and; T-P concentration in the pond was 0.08, 0.1 and 0.09 mg T-PL<sup>-1</sup> (semi-saturation constant 0.056 mg T-PL<sup>-1</sup>) in May, August and October respectively. Limiting function numbers were 0.90 for light and temperature and 0.52 for nutrient. The model simulated the observed biomass (4.82 kg) with no apparent discrepancy and closely reproducing the growth trend (Figure 3a). In the same manner, respective temperature for Purple loosestrife in 2007 was 23.5, 27.7 and 17.2°C in May, August and October; light intensity was 388.8, 416.6 and 250.0 Wm<sup>-2</sup> and; concentration of T-P in the pond

was 0.1, 0.94 and 0.2 mg T-PL<sup>-1</sup> in the same period. Limiting function number for temperature, light and nutrient were 0.78, 0.87 and 0.32 respectively. The observed biomass (7.39 kg) was simulated by the model with 2.43% discrepancy (Figure 3b).



**Figure 3** Comparison of simulated and observed biomass under long term cultivation test (a) Purple loosestrife in 2006, (b) Purple loosestrife in 2007, (c) Common reed in 2006, (d) Common reed in 2007 and, (e) an illustration of total biomass harvest against different level of biomass maintenance (B<sub>max</sub> is considered as maximum biomass/m<sup>2</sup> produced by the macrophytes under natural condition)

Environmental conditions for Common reed in 2006 was temperature of 22, 27, and 18.3<sup>0</sup>C in May, August and October (optimal 25<sup>0</sup>C); light intensity of 361.1, 388.8, and 305.5 Wm<sup>-2</sup> (optimal 411.0 Wm<sup>-2</sup>) and; T-P in the pond of 0.09, 0.1 and 0.09 mg T-PL<sup>-1</sup> (semi-saturation constant 0.02 mg T-PL<sup>-1</sup>) in the same period respectively. Respective limiting function numbers for temperature, light and nutrient were 0.96, 0.89 and 0.61. Discrepancy in model simulated biomass was 8.5% against observed biomass (10.87 kg) and the growth trend was close to the observed growth trend (Figure 3c). Again for Common reed in 2007, temperature was 23.5, 27.7 and 11<sup>0</sup>C; light intensity was 388.88, 416.6 and 250.0 Wm<sup>-2</sup> and; T-P in the pond was 0.09, 0.09 and 0.2 mg T-PL<sup>-1</sup> in May, August and November respectively. Limiting function numbers for temperature and light were 0.86 and for nutrient was 0.78. The model simulated the observed biomass (13.67 kg) with 11.41% discrepancy and similar growth trend (Figure 3d). Most discrepancies in model simulated biomass can be attributed to the inability of the nutrient uptake functions (lower limiting function numbers) to sufficiently reproduce the lower nutrient conditions of water environment. This is because if nutrient concentration of water environment is close to the semi-saturation constants of nutrients of test plants, the limiting function number for nutrient is close to unity thereby increasing performance of the model. Approximation of biomass remained at the experimental base after harvest in case of emergent macrophytes can also be attributed to discrepancies in model performance as it was not possible to weigh the biomass like in floating macrophytes. However, model simulation results corroborate with many growth and biomass models of like Asaeda *et al.* 2000 and 2001 indicating ±12% discrepancies in model simulations are acceptable.

The model was applied to simulate total biomass production at different levels of biomass maintenance against maximum biomass ( $B_{\max}$ ) of macrophytes using input data of 2007 of the experimental pond (Figure 2e and 3e). The simulation revealed that the total biomass produced by Giant duckweed was significantly higher than rest of the macrophytes due to its high growth rate feature. Moreover, for all the macrophytes maximum total biomass was obtained when the biomass level was maintained at 50-60% of  $B_{\max}$ .

#### *Reliability of parameters*

Most growth parameters of the model were measured and their reliability is likely to be higher than estimated mortality parameters. However the model simulated results obtained using present set of parameters was satisfactory. In the present situation, two years observed data of Purple loosestrife and Common reed (2006-2007) and one year data of Water hyacinth (2005) and Giant duckweed (2006 laboratory based and 2007 field based) is relatively insufficient to check reliability of parameters. So observed data of 3-4 years is needed for more robustness of parameters and model.

### **CONCLUSIONS**

The biomass production model was formulated. Its simulation results were within  $\pm 12\%$  discrepancies suggesting a good level of modeling by incorporating only environmental growth factors in the model of this study. However, the model needs more tests to increase reliability of parameters and robustness of model structure. Giant duckweed showed its potential to grow well under wide range of light intensities and significantly yielded high biomass at different levels of biomass maintenance. Models as simple as introduced in this study, measurable parameters and mostly accessible environmental input data from public domains make it possible for wider application. So the model can be applied to other macrophytes. The model has scope for the environmental engineers to advance research into environmental factors that regulate growth and predict growth and biomass changes over time under different environmental scenarios. For lake and pond management the model is simple especially to prepare biomass harvesting schedules for the maintenance of biomass at optimum level.

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