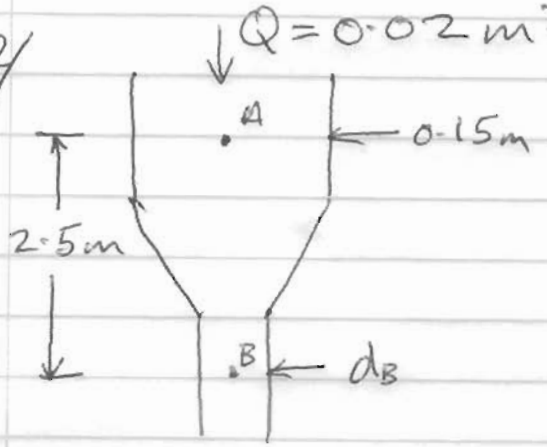
Solutions for CIVE1400 exam June 2007

Q1/

See notes.
Half page description plus diagrams for each item.

Q2/



$$A_A = \frac{\pi \times 0.15^2}{4} = 0.01768$$

$$P_A - P_B = -22000 \text{ N/m}^2$$
$$Z_A - Z_B = 2.5\text{m}$$

$$U_A = \frac{Q}{A_A} = \frac{0.02}{A_A} = 1.1318 \text{ m/s}$$

$$\frac{P_A}{\rho g} + \frac{U_A^2}{2g} + Z_A = \frac{P_B}{\rho g} + \frac{U_B^2}{2g} + Z_B$$

$$\frac{P_A - P_B}{\rho g} + (Z_A - Z_B) + \frac{U_A^2}{2g} = \frac{U_B^2}{2g}$$

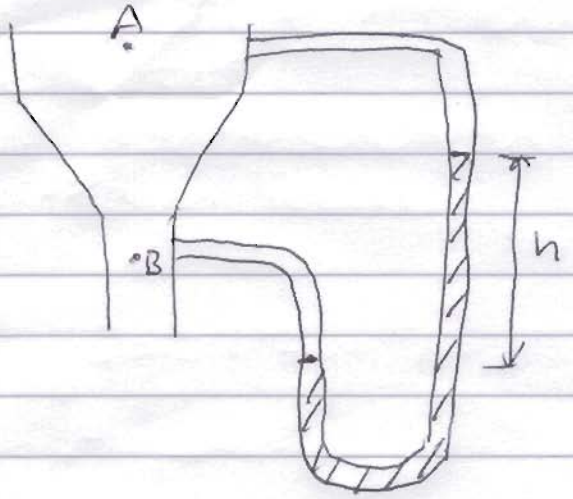
$$(-2.243 + 2.5 + 0.0653)\,19.62 = U_B^2$$
$$U_B^2 = 6.33$$
$$U_B = 2.516 \text{ m/s}$$

$$A_B = \frac{0.02}{U_B} = 0.0079 \text{ m}^2$$

$$d_B = \sqrt{\frac{4 \times 0.0079}{\pi}} = 0.1\text{m}$$

P2

Q2b

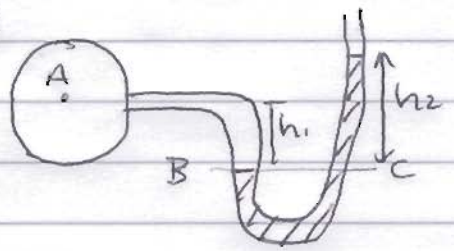


$$\frac{P_B - P_A}{\rho_{manog}} = h$$

$$\frac{22000}{13600 \times 9.81} = 0.165m$$

$h = 0.165m.$

Q2 a)



$P_{ATMOS} = 101.33\ kN/m^2$

$P_A = 250\ kN/m^2$

$h_1 = 1.5m$

$P_B = P_C$

$$P_A + h_1 \rho g = P_{ATMOS} + h_2 \rho_{man} g$$

$$250000 + 1.5 \times 1000 \times 9.81 = 101330 + h_2 13600 \times 9.81$$

$$h_2 = 1.22m.$$

b) $$R = \rho g h \times \frac{h}{2} \times L = 800 \times 9.81 \times \frac{2^2}{2} \times L$$

$R = 15696\ L \quad N/m$

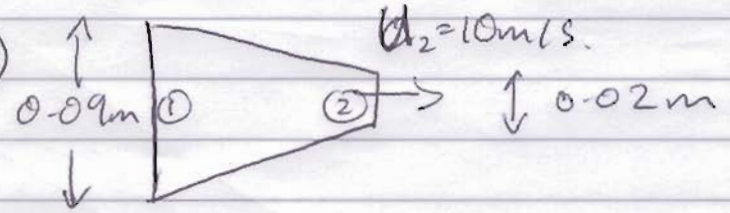
$R_{1.5} = 23544\ N$, $R_{2.5} = 39240\ N.$

Position $D = \frac{2}{3}h = \frac{2}{3} \times 2 = 1\frac{1}{3}$ from surface.

p3

Q3

c.)



$$\frac{P_1}{\rho g} + \frac{U_1^2}{2g} + \cancel{Z_1} = \frac{P_2}{\rho g} + \frac{U_2^2}{2g} + \cancel{Z_2}$$

P_2 = Atmospheric, P_1 = gauge.

$$\frac{P_1}{\rho g} = \frac{U_2^2 - U_1^2}{2g}$$

$$U_1 = \frac{A_2 U_2}{A_1} = 0.4938 \text{ m/s}$$

$$P_1 = \frac{\rho}{2} U_2^2 \left(1 - \frac{A_2^2}{A_1^2}\right)$$

$$P_1 \approx 49\,878 \text{ N}$$

p4.

Q4

$A_1 = \frac{\pi d_1^2}{4} = 0.038\ m^2$ $\quad U_1 = \frac{Q}{A_1} = 0.526\ m/s$

$A_2 = \frac{\pi d_2^2}{4} = 0.096\ m^2$ $\quad U_2 = \frac{Q}{A_2} = 0.2079\ m/s$

$Q = 0.02\ m^3/s$ $\quad , \theta_1 = 0 , \theta_2 = 180^\circ$

$P_2 = 20000\ N/m^2$

Total force

$F_{Tx} = \rho Q(U_2 \cos\theta_2 - U_1) = -14.68\ N$

$F_{Ty} = \rho g(U_2 \sin\theta_2 - U_1 \sin 0) = 0.$

Pressure force

$P_1 = P_2 + \frac{\rho}{2}(U_2^2 - U_1^2) + (z_1 - z_2)$ (z terms $= 0$)

$P_1 = 19883\ N/m^2$

$F_{Px} = P_1 A_1 \cos 0 - P_2 A_2 \cos\theta_2 = 2680\ N$

$F_{Py} = P_1 A_1 \sin 0 - P_2 A_2 \sin\theta_2 = 0\ N.$

Body force.

In horizontal plane so no body forces

$F_{Bx} = F_{By} = 0$

Resultant force.

$F_{Tx} = F_{Rx} + F_{Px} + F_{Bx}$

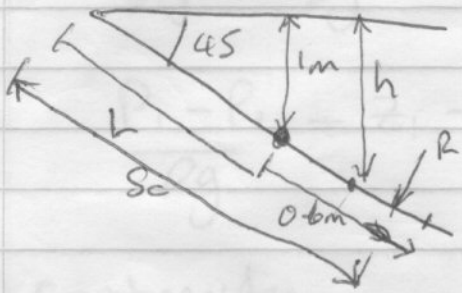
$F_{Rx} = -2694\ N.$

$F_{Ry} = 0$

Force on bend $= -F_{Rx} = 2694\ N$ @ 0°

ie horizontal.

Q5



$$A = \frac{\pi d^2}{4} = \frac{\pi \times 0.6^2}{4} = 0.28 m^2$$

$$r = \frac{0.6}{2} = 0.3 m.$$

a) R = Pressure at centroid × Area.

centroid $L \cos 45 = 1$

$L = 1.4142 m.$

$h = 1.7142 \quad \cos 45 = 1.212 m$

$R = \rho g h A$

$=$

b) $I_{GG} = \frac{\pi r^4}{4}$

$I_{OO} = I_{GG} + A\bar{x}^2$

$\bar{x} = 1.412 + 0.3 = 1.712 m.$

$S_c = \left(\frac{\pi r^4}{4} + A\bar{x}^2 \right) / A\bar{x}$

$S_c = 1.725 m$

c) $m = R(S_c - 1.4121) = 042 \quad Nm.$

d) As the level rises the centre of pressure rises towards the position of the centroid.

Q6

$$\frac{P_1}{\rho g} + \frac{u_1^2}{2g} + z_1 = \frac{P_2}{2g} + \frac{u_2^2}{2g} + z_2$$

$$\frac{P_1 - P_2}{\rho g} + z_1 - z_2 = \frac{u_2^2 - u_1^2}{2g}$$

Continuity

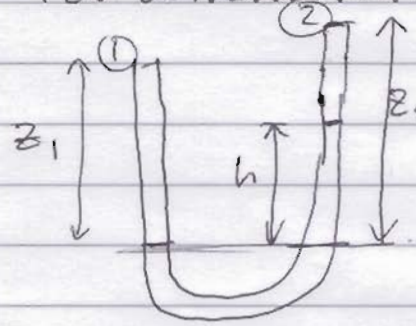
$$Q = u_1 A_1 = u_2 A_2$$

$$u_2 = \frac{A_1}{A_2} u_1$$

then

$$\frac{P_1 - P_2}{\rho g} + z_1 - z_2 = \frac{u_1^2}{2g}\left(\frac{A_1^2}{A_2^2} - 1\right) = \frac{u_1^2}{2g}\left(\frac{A_1^2 - A_2^2}{A_2^2}\right)$$

For a manometer



$$P_1 + \rho g z_1 = P_2 + \rho_{man} g h + \rho g (z_2 - h)$$

$$\frac{P_1 - P_2}{\rho g} + z_1 - z_2 = h\left(\frac{\rho_{man}}{\rho} - 1\right)$$

$\therefore$

$$\frac{u_1^2}{2g}\left(\frac{A_1^2 - A_2^2}{A_2^2}\right) = h\left(\frac{\rho_{man}}{\rho} - 1\right)$$

$$u_1 = A_2 \sqrt{\frac{2gh\left(\rho_{man}/\rho - 1\right)}{A_1^2 - A_2^2}}$$

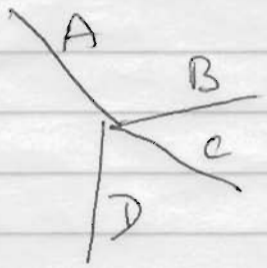
$Q_{theoretical} = u_1 A_1$

$$Q = C_d A_1 A_2 \sqrt{\frac{2gh\left(\rho_{man}/\rho - 1\right)}{A_1^2 - A_2^2}}$$

Qed.

P7

Q7



$d_A = ?$ $\quad A_A = ?$

$d_B = 0.05m$ $\quad A_B = 0.00196m^2$

$d_C = 0.1m$ $\quad A_C = 0.00785m^2$

$d_d = 0.15m$ $\quad A_D = 0.01767m^2$

$U_A = 1.0 m/s$

$V_B = Q_B / A_B = 0.643 \ m/s$

$V_C = Q_C / A_C = 0.321 \ m/s$

$V_d = 0.5 m/s$

$Q_A = 0.01262 \ m^3/s$

$Q_B = 0.1 Q_A = 0.00126 m^3/s$

$Q_C = 0.2 Q_A = 0.00252 m^3/s$

$Q_D = 0.0088 \ m^3/s$

$Q_D = Q_A 0.7 = A_D V_D = 0.0088 m^3/s$

$Q_A = \frac{Q_D}{0.7} = 0.01262 \ m^3/s$

$Q_B = 0.1 Q_A = 0.00126 \ m^3/s$

$Q_C = 0.2 Q_A = 0.00252 \ m^3/s$

$$\frac{P_1 - P_2}{\rho g} + \frac{v_1^2 - v_2^2}{2g} + z_1 = z_2$$

$v_1 = v_2$; $\quad \frac{100\,000}{9.81 \times 1000} + 100 = z_2 = 110.19m$